## Supplementary Material for "Rewarded Semi-Supervised Re-Identification on Identities Rarely Crossing Camera Views"

Ancong Wu, Wenhang Ge, Wei-Shi Zheng

Abstract—This supplementary material contains detailed description of the training algorithm and more visualizations of entropy distribution and learned probabilistic relations for comprehensively understanding our method rewarded relation discovery (R<sup>2</sup>D).

## **1** TRAINING ALGORITHM

The training algorithm of our Rewarded Relation Discovery ( $R^2D$ ) is shown in Algorithm 1.

Input: unlabelled data set $\mathcal{D}_{U}$ , labelled data set $\mathcal{D}_{L}$ Output: model $F(\cdot; \Theta^{(t_{max})})$ Require: label smooth parameter $\lambda$ , step sizes $\alpha, \gamma_{R}, \gamma_{\theta}, \gamma_{p}$ , maximum iteration number $t_{max}$ 1 Initialize model parameters $\Theta^{(0)}$ by pretraining 2 Initialize cluster relation matrix $\hat{\mathbf{R}}^{dyn(0)}$ by Eq. (3) 3 $t = 0$ 4 while $t \leq t_{max}$ do 5 Sample batch $\mathcal{B}_{U}$ and $\mathcal{B}_{L}$ from $\mathcal{D}_{U}$ and $\mathcal{D}_{L}$ , respectively 6 Compute dynamic cluster discrimination loss $\mathcal{L}_{cd}^{dyn}(\mathcal{B}_{U}; \hat{\mathbf{R}}^{dyn}, \Theta, \{\mathbf{p}_{c}\})$ in Eq. (4) 7 Approximate optimal model parameter $\Theta'$ and prototype parameter $\mathbf{p}_{c}'$ by Eq. (6) and Eq. (7) $\Theta' = \Theta^{(t)} - \alpha \frac{\partial \mathcal{L}_{cd}^{dyn}(\mathcal{B}_{U}; \hat{\mathbf{R}}^{dyn(t)}, \Theta^{(t)}, \{\mathbf{p}_{c}^{(t)}\})}{\partial \mathbf{p}_{c}^{(t)}}$ 8 Compute identification loss $\mathcal{L}_{id}(\mathcal{B}_{L}; \Theta, \{\mathbf{p}_{c}\})$ in Eq. (5) 9 Update cluster relation matrix $\hat{\mathbf{R}}^{dyn}$ by Eq. (8) $\hat{\mathbf{R}}^{dyn(t+1)} = \hat{\mathbf{R}}^{dyn(t)} - \gamma_{R} \frac{\partial \mathcal{L}_{id}(\mathcal{B}_{L}; \Theta', \{\mathbf{p}_{c}\})}{\partial \hat{\mathbf{R}}^{dyn(t)}}$ 10 Update model parameter $\Theta$ and prototype $\mathbf{p}_{c}$ with updated cluster relation matrix $\hat{\mathbf{R}}^{dyn}(\mathbf{k}) = \mathbf{p}_{c}^{(t+1)} = \Theta^{(t)} - \gamma_{\theta} \frac{\partial \mathcal{L}_{cd}^{dyn}(\mathcal{B}_{U}; \hat{\mathbf{R}}^{dyn(t+1)}, \Theta^{(t)}, \{\mathbf{p}_{c}^{(t)}\})}{\partial \hat{\mathbf{R}}^{dyn(t)}}$ $\mathbf{P}_{c}^{(t+1)} = \mathbf{P}^{(t)} - \gamma_{\theta} \frac{\partial \mathcal{L}_{cd}^{dyn}(\mathcal{B}_{U}; \hat{\mathbf{R}}^{dyn(t+1)}, \Theta^{(t)}, \{\mathbf{p}_{c}^{(t)}\})}{\partial \mathbf{p}_{c}^{(t)}}}$ $\mathbf{P}_{c}^{(t+1)} = \mathbf{P}_{c}^{(t)} - \gamma_{\theta} \frac{\partial \mathcal{L}_{cd}^{dyn}(\mathcal{B}_{U}; \hat{\mathbf{R}}^{dyn(t+1)}, \Theta^{(t)}, \{\mathbf{p}_{c}^{(t)}\})}{\partial \mathbf{p}_{c}^{(t)}}}$ $\mathbf{P}_{c}^{(t+1)} = \mathbf{P}_{c}^{(t)} - \gamma_{\theta} \frac{\partial \mathcal{L}_{cd}^{dyn}(\mathcal{B}_{U}; \hat{\mathbf{R}}^{dyn(t+1)}, \Theta^{(t)}, \{\mathbf{p}_{c}^{(t)}\})}{\partial \mathbf{p}_{c}^{(t)}}}$ $\mathbf{P}_{c}^{(t+1)} = \mathbf{P}_{c}^{(t)} - \gamma_{\theta} \frac{\partial \mathcal{L}_{cd}^{dyn}(\mathcal{B}_{U}; \hat{\mathbf{R}}^{dyn(t+1)}, \Theta^{(t)}, \{\mathbf{p}_{c}^{(t)}\})}{\partial \mathbf{p}_{c}^{(t)}}}$ $\mathbf{P}_{c}^{(t+1)} = \mathbf{P}_{c}^{(t)} - \gamma_{\theta} \frac{\partial \mathcal{L}_{cd}^{dyn}(\mathcal{B}_{U}; \hat{\mathbf{R}}^{dyn(t+1)}, \Theta^{(t)}, \{\mathbf{p}_{c}^{(t)}\})}{\partial \mathbf{p}_{c}^{(t)}}}$ $\mathbf{P}_{c}^{(t+1)} = \mathbf{P}_{c}^{(t)} -$	Algorithm 1: Rewarded Relation Discovery
Output: model $F(\cdot; \Theta^{(t_{max})})$ Require: label smooth parameter $\lambda$ , step sizes $\alpha, \gamma_R, \gamma_\theta, \gamma_p$ , maximum iteration number $t_{max}$ 1 Initialize model parameters $\Theta^{(0)}$ by pretraining 2 Initialize cluster relation matrix $\hat{\mathbf{R}}^{dyn(0)}$ by Eq. (3) 3 $t = 0$ 4 while $t \leq t_{max}$ do 5 Sample batch $\mathcal{B}_U$ and $\mathcal{B}_L$ from $\mathcal{D}_U$ and $\mathcal{D}_L$ , respectively 6 Compute dynamic cluster discrimination loss $\mathcal{L}_{cd}^{dyn}(\mathcal{B}_U; \hat{\mathbf{R}}^{dyn}, \Theta, \{\mathbf{p}_c\})$ in Eq. (4) 7 Approximate optimal model parameter $\Theta'$ and prototype parameter $\mathbf{p}'_c$ by Eq. (6) and Eq. (7) $\Theta' = \Theta^{(t)} - \alpha \frac{\partial \mathcal{L}_{cd}^{dyn}(\mathcal{B}_U; \hat{\mathbf{R}}^{dyn(t)}, \Theta^{(t)}, \{\mathbf{p}_c^{(t)}\})}{\partial \mathbf{p}_c^{(t)}}$ 8 Compute identification loss $\mathcal{L}_{id}(\mathcal{B}_L; \Theta, \{\mathbf{p}_c\})$ in Eq. (5) 9 Update cluster relation matrix $\hat{\mathbf{R}}^{dyn}$ by Eq. (8) $\hat{\mathbf{R}}^{dyn(t+1)} = \hat{\mathbf{R}}^{dyn(t)} - \gamma_R \frac{\partial \mathcal{L}_{id}(\mathcal{B}_L; \Theta', \{\mathbf{p}_c\})}{\partial \hat{\mathbf{R}}^{dyn(t)}}$ 10 Update model parameter $\Theta$ and prototype $\mathbf{p}_c$ with updated cluster relation matrix $\hat{\mathbf{R}}^{dyn}(\mathbf{k})$ 10 $\Theta^{(t+1)} = \Theta^{(t)} - \gamma_{\theta} \frac{\partial \mathcal{L}_{cd}^{dyn}(\mathcal{B}_U; \hat{\mathbf{R}}^{dyn(t+1)}, \Theta^{(t)}, \{\mathbf{p}_c^{(t)}\})}{\partial \Theta^{(t)}}$ $\mathbf{p}_c^{(t+1)} = \mathbf{p}_c^{(t)} - \gamma_{\theta} \frac{\partial \mathcal{L}_{cd}^{dyn}(\mathcal{B}_U; \hat{\mathbf{R}}^{dyn(t+1)}, \Theta^{(t)}, \{\mathbf{p}_c^{(t)}\})}{\partial \mathbf{p}_c^{(t)}}}$ 11 $t \leftarrow t + 1$ 12 end	<b>Input:</b> unlabelled data set $\mathcal{D}_U$ , labelled data set $\mathcal{D}_L$
<b>Require</b> : label smooth parameter $\lambda$ , step sizes $\alpha, \gamma_R, \gamma_\theta, \gamma_p$ , maximum iteration number $t_{max}$ 1 Initialize model parameters $\Theta^{(0)}$ by pretraining 2 Initialize cluster relation matrix $\hat{\mathbf{R}}^{dyn(0)}$ by Eq. (3) 3 $t = 0$ 4 while $t \leq t_{max}$ do 5 Sample batch $\mathcal{B}_U$ and $\mathcal{B}_L$ from $\mathcal{D}_U$ and $\mathcal{D}_L$ , respectively 6 Compute dynamic cluster discrimination loss $\mathcal{L}_{cd}^{dyn}(\mathcal{B}_U; \hat{\mathbf{R}}^{dyn}, \Theta, \{\mathbf{p}_c\})$ in Eq. (4) 7 Approximate optimal model parameter $\Theta'$ and prototype parameter $\mathbf{p}'_c$ by Eq. (6) and Eq. (7) $\Theta' = \Theta^{(t)} - \alpha \frac{\partial \mathcal{L}_{cd}^{dyn}(\mathcal{B}_U; \hat{\mathbf{R}}^{dyn(t)}, \Theta^{(t)}, \{\mathbf{p}_c^{(t)}\})}{\partial \mathbf{p}_c^{(t)}}$ 8 Compute identification loss $\mathcal{L}_{id}(\mathcal{B}_L; \Theta, \{\mathbf{p}_c\})$ in Eq. (5) 9 Update cluster relation matrix $\hat{\mathbf{R}}^{dyn}$ by Eq. (8) $\hat{\mathbf{R}}^{dyn(t+1)} = \hat{\mathbf{R}}^{dyn(t)} - \gamma_R \frac{\partial \mathcal{L}_{id}(\mathcal{B}_L; \Theta', \{\mathbf{p}_c\})}{\partial \hat{\mathbf{R}}^{dyn(t)}}$ 10 Update model parameter $\Theta$ and prototype $\mathbf{p}_c$ with updated cluster relation matrix $\hat{\mathbf{R}}^{dyn}$ by Eq. (8) $\hat{\mathbf{R}}^{dyn(t+1)} = \Theta^{(t)} - \gamma_\theta \frac{\partial \mathcal{L}_{cd}^{dyn}(\mathcal{B}_U; \hat{\mathbf{R}}^{dyn(t+1)}, \Theta^{(t)}, \{\mathbf{p}_c^{(t)}\})}{\partial \hat{\mathbf{R}}^{dyn(t)}}$ 10 $\Theta^{(t+1)} = \Theta^{(t)} - \gamma_\theta \frac{\partial \mathcal{L}_{cd}^{dyn}(\mathcal{B}_U; \hat{\mathbf{R}}^{dyn(t+1)}, \Theta^{(t)}, \{\mathbf{p}_c^{(t)}\})}{\partial \Theta^{(t)}}$ $\mathbf{p}_c^{(t+1)} = \mathbf{p}_c^{(t)} - \gamma_p \frac{\partial \mathcal{L}_{cd}^{dyn}(\mathcal{B}_U; \hat{\mathbf{R}}^{dyn(t+1)}, \Theta^{(t)}, \{\mathbf{p}_c^{(t)}\})}{\partial \mathbf{p}_c^{(t)}}}$ 11 $t \leftarrow t + 1$ 12 end	<b>Output:</b> model $F(\cdot; \Theta^{(t_{max})})$
$\begin{array}{ll} \alpha, \gamma_{R}, \gamma_{\theta}, \gamma_{p}, \mbox{ maximum iteration number } t_{max} \\ 1 \mbox{ Initialize model parameters } \Theta^{(0)} \mbox{ by pretraining} \\ 2 \mbox{ Initialize cluster relation matrix } \hat{\mathbf{R}}^{dyn(0)} \mbox{ by Eq. (3)} \\ 3 \ t = 0 \\ 4 \ \mbox{ while } t \leq t_{max} \ \mbox{ do} \\ 5 \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \$	<b>Require</b> : label smooth parameter $\lambda$ , step sizes
1 Initialize model parameters $\Theta^{(0)}$ by pretraining 2 Initialize cluster relation matrix $\hat{\mathbf{R}}^{dyn(0)}$ by Eq. (3) 3 $t = 0$ 4 while $t \leq t_{max}$ do 5 Sample batch $\mathcal{B}_U$ and $\mathcal{B}_L$ from $\mathcal{D}_U$ and $\mathcal{D}_L$ , respectively 6 Compute dynamic cluster discrimination loss $\mathcal{L}_{cd}^{dyn}(\mathcal{B}_U; \hat{\mathbf{R}}^{dyn}, \Theta, \{\mathbf{p}_c\})$ in Eq. (4) 7 Approximate optimal model parameter $\Theta'$ and prototype parameter $\mathbf{p}'_c$ by Eq. (6) and Eq. (7) $\Theta' = \Theta^{(t)} - \alpha \frac{\partial \mathcal{L}_{cd}^{dyn}(\mathcal{B}_U; \hat{\mathbf{R}}^{dyn(t)}, \Theta^{(t)}, \{\mathbf{p}_c^{(t)}\})}{\partial \mathbf{p}_c^{(t)}}$ 8 Compute identification loss $\mathcal{L}_{id}(\mathcal{B}_L; \Theta, \{\mathbf{p}_c\})$ in Eq. (5) 9 Update cluster relation matrix $\hat{\mathbf{R}}^{dyn}$ by Eq. (8) $\hat{\mathbf{R}}^{dyn(t+1)} = \hat{\mathbf{R}}^{dyn(t)} - \gamma_R \frac{\partial \mathcal{L}_{id}(\mathcal{B}_L; \Theta', \{\mathbf{p}_c\})}{\partial \hat{\mathbf{R}}^{dyn(t)}}$ 10 Update model parameter $\Theta$ and prototype $\mathbf{p}_c$ with updated cluster relation matrix $\hat{\mathbf{R}}^{dyn(t+1)}, \Theta^{(t)}, \{\mathbf{p}_c^{(t)}\})$ $\Theta^{(t+1)} = \Theta^{(t)} - \gamma_{\theta} \frac{\partial \mathcal{L}_{cd}^{dyn}(\mathcal{B}_U; \hat{\mathbf{R}}^{dyn(t+1)}, \Theta^{(t)}, \{\mathbf{p}_c^{(t)}\})}{\partial \Theta^{(t)}}$ $\mathbf{p}_c^{(t+1)} = \mathbf{p}_c^{(t)} - \gamma_p \frac{\partial \mathcal{L}_{cd}^{dyn}(\mathcal{B}_U; \hat{\mathbf{R}}^{dyn(t+1)}, \Theta^{(t)}, \{\mathbf{p}_c^{(t)}\})}{\partial \mathbf{p}_c^{(t)}}}$ 11 $t \leftarrow t + 1$ 12 end	$\alpha, \gamma_R, \gamma_{\theta}, \gamma_p$ , maximum iteration number $t_{max}$
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6 Compute dynamic cluster discrimination loss $\mathcal{L}_{cd}^{dyn}(\mathcal{B}_{U}; \hat{\mathbf{R}}^{dyn}, \Theta, \{\mathbf{p}_{c}\}) \text{ in Eq. (4)}$ 7 Approximate optimal model parameter $\Theta'$ and prototype parameter $\mathbf{p}_{c}'$ by Eq. (6) and Eq. (7) $\Theta' = \Theta^{(t)} - \alpha \frac{\partial \mathcal{L}_{cd}^{dyn}(\mathcal{B}_{U}; \hat{\mathbf{R}}^{dyn(t)}, \Theta^{(t)}, \{\mathbf{p}_{c}^{(t)}\})}{\partial \Theta^{(t)}}$ $\mathbf{p}_{c}' = \mathbf{p}_{c}^{(t)} - \alpha \frac{\partial \mathcal{L}_{cd}^{dyn}(\mathcal{B}_{U}; \hat{\mathbf{R}}^{dyn(t)}, \Theta^{(t)}, \{\mathbf{p}_{c}^{(t)}\})}{\partial \mathbf{p}_{c}^{(t)}}$ 8 Compute identification loss $\mathcal{L}_{id}(\mathcal{B}_{L}; \Theta, \{\mathbf{p}_{c}\})$ in Eq. (5) 9 Update cluster relation matrix $\hat{\mathbf{R}}^{dyn}$ by Eq. (8) $\hat{\mathbf{R}}^{dyn(t+1)} = \hat{\mathbf{R}}^{dyn(t)} - \gamma_{R} \frac{\partial \mathcal{L}_{id}(\mathcal{B}_{L}; \Theta', \{\mathbf{p}_{c}'\})}{\partial \hat{\mathbf{R}}^{dyn(t)}}$ 10 Update model parameter $\Theta$ and prototype $\mathbf{p}_{c}$ with updated cluster relation matrix $\hat{\mathbf{R}}^{dyn(t+1)}$ by Eq. (9) and Eq. (10) $\Theta^{(t+1)} = \Theta^{(t)} - \gamma_{\theta} \frac{\partial \mathcal{L}_{cd}^{dyn}(\mathcal{B}_{U}; \hat{\mathbf{R}}^{dyn(t+1)}, \Theta^{(t)}, \{\mathbf{p}_{c}^{(t)}\})}{\partial \Theta^{(t)}}$ $\mathbf{p}_{c}^{(t+1)} = \mathbf{p}_{c}^{(t)} - \gamma_{p} \frac{\partial \mathcal{L}_{cd}^{dyn}(\mathcal{B}_{U}; \hat{\mathbf{R}}^{dyn(t+1)}, \Theta^{(t)}, \{\mathbf{p}_{c}^{(t)}\})}}{\partial \mathbf{p}_{c}^{(t)}}$ 11 $t \leftarrow t + 1$ 12 end	respectively
$ \begin{array}{l} \mathcal{L}_{cd}^{ayn}(\mathcal{B}_{U};\hat{\mathbf{R}}^{dyn},\Theta,\{\mathbf{p}_{c}\}) \text{ in Eq. (4)} \\ \text{Approximate optimal model parameter } \boldsymbol{\Theta}' \text{ and} \\ \text{prototype parameter } \mathbf{p}_{c}' \text{ by Eq. (6) and Eq. (7)} \\ \boldsymbol{\Theta}' = \boldsymbol{\Theta}^{(t)} - \alpha \frac{\partial \mathcal{L}_{cd}^{dyn}(\mathcal{B}_{U};\hat{\mathbf{R}}^{dyn(t)},\Theta^{(t)},\{\mathbf{p}_{c}^{(t)}\})}{\partial \mathbf{\Theta}^{(t)}} \\ \mathbf{p}_{c}' = \mathbf{p}_{c}^{(t)} - \alpha \frac{\partial \mathcal{L}_{cd}^{dyn}(\mathcal{B}_{U};\hat{\mathbf{R}}^{dyn(t)},\Theta^{(t)},\{\mathbf{p}_{c}^{(t)}\})}{\partial \mathbf{p}_{c}^{(t)}} \\ \text{8 Compute identification loss } \mathcal{L}_{id}(\mathcal{B}_{L};\Theta,\{\mathbf{p}_{c}\}) \text{ in Eq. (5)} \\ \text{9 Update cluster relation matrix } \hat{\mathbf{R}}^{dyn} \text{ by Eq. (8)} \\ \hat{\mathbf{R}}^{dyn(t+1)} = \hat{\mathbf{R}}^{dyn(t)} - \gamma_{R} \frac{\partial \mathcal{L}_{id}(\mathcal{B}_{L};\Theta',\{\mathbf{p}_{c}'\})}{\partial \hat{\mathbf{R}}^{dyn(t)}} \\ \text{Update model parameter } \boldsymbol{\Theta} \text{ and prototype } \mathbf{p}_{c} \\ \text{with updated cluster relation matrix } \hat{\mathbf{R}}^{dyn(t+1)}, \mathbf{\Theta}^{(t)},\{\mathbf{p}_{c}^{(t)}\}) \\ \mathbf{\Theta}^{(t+1)} = \mathbf{\Theta}^{(t)} - \gamma_{\theta} \frac{\partial \mathcal{L}_{cd}^{dyn}(\mathcal{B}_{U};\hat{\mathbf{R}}^{dyn(t+1)},\mathbf{\Theta}^{(t)},\{\mathbf{p}_{c}^{(t)}\})}{\partial \mathbf{p}_{c}^{(t)}} \\ \mathbf{p}_{c}^{(t+1)} = \mathbf{p}_{c}^{(t)} - \gamma_{p} \frac{\partial \mathcal{L}_{cd}^{dyn}(\mathcal{B}_{U};\hat{\mathbf{R}}^{dyn(t+1)},\mathbf{\Theta}^{(t)},\{\mathbf{p}_{c}^{(t)}\})}{\partial \mathbf{p}_{c}^{(t)}} \\ 10 \text{ t} \leftarrow t+1 \\ \text{12 end} \end{array}$	6 Compute dynamic cluster discrimination loss
7 Approximate optimal model parameter $\Theta'$ and prototype parameter $\mathbf{p}_{c}'$ by Eq. (6) and Eq. (7) $\Theta' = \Theta^{(t)} - \alpha \frac{\partial \mathcal{L}_{cd}^{dyn}(\mathcal{B}_{U}; \hat{\mathbf{R}}^{dyn(t)}, \Theta^{(t)}, \{\mathbf{p}_{c}^{(t)}\})}{\partial \mathbf{p}_{c}^{(t)}}$ $\mathbf{p}_{c}' = \mathbf{p}_{c}^{(t)} - \alpha \frac{\partial \mathcal{L}_{cd}^{dyn}(\mathcal{B}_{U}; \hat{\mathbf{R}}^{dyn(t)}, \Theta^{(t)}, \{\mathbf{p}_{c}^{(t)}\})}{\partial \mathbf{p}_{c}^{(t)}}$ 8 Compute identification loss $\mathcal{L}_{id}(\mathcal{B}_{L}; \Theta, \{\mathbf{p}_{c}\})$ in Eq. (5) 9 Update cluster relation matrix $\hat{\mathbf{R}}^{dyn}$ by Eq. (8) $\hat{\mathbf{R}}^{dyn(t+1)} = \hat{\mathbf{R}}^{dyn(t)} - \gamma_{R} \frac{\partial \mathcal{L}_{id}(\mathcal{B}_{L}; \Theta', \{\mathbf{p}_{c}'\})}{\partial \hat{\mathbf{R}}^{dyn(t)}}$ 10 Update model parameter $\Theta$ and prototype $\mathbf{p}_{c}$ with updated cluster relation matrix $\hat{\mathbf{R}}^{dyn(t+1)}$ by Eq. (9) and Eq. (10) $\Theta^{(t+1)} = \Theta^{(t)} - \gamma_{\theta} \frac{\partial \mathcal{L}_{cd}^{dyn}(\mathcal{B}_{U}; \hat{\mathbf{R}}^{dyn(t+1)}, \Theta^{(t)}, \{\mathbf{p}_{c}^{(t)}\})}{\partial \mathbf{p}_{c}^{(t)}}$ $\mathbf{p}_{c}^{(t+1)} = \mathbf{p}_{c}^{(t)} - \gamma_{p} \frac{\partial \mathcal{L}_{cd}^{dyn}(\mathcal{B}_{U}; \hat{\mathbf{R}}^{dyn(t+1)}, \Theta^{(t)}, \{\mathbf{p}_{c}^{(t)}\})}{\partial \mathbf{p}_{c}^{(t)}}$ 11 $t \leftarrow t + 1$ 12 end	$\mathcal{L}_{cd}^{ayn}(\mathcal{B}_U;\mathbf{\hat{R}}^{dyn},\mathbf{\Theta},\{\mathbf{p}_c\})$ in Eq. (4)
prototype parameter $\mathbf{p}_{c}'$ by Eq. (6) and Eq. (7) $\mathbf{\Theta}' = \mathbf{\Theta}^{(t)} - \alpha \frac{\partial \mathcal{L}_{cd}^{dyn}(\mathcal{B}_{U}; \hat{\mathbf{R}}^{dyn(t)}, \mathbf{\Theta}^{(t)}, \{\mathbf{p}_{c}^{(t)}\})}{\partial \mathbf{\Theta}^{(t)}}$ $\mathbf{p}_{c}' = \mathbf{p}_{c}^{(t)} - \alpha \frac{\partial \mathcal{L}_{cd}^{dyn}(\mathcal{B}_{U}; \hat{\mathbf{R}}^{dyn(t)}, \mathbf{\Theta}^{(t)}, \{\mathbf{p}_{c}^{(t)}\})}{\partial \mathbf{p}_{c}^{(t)}}$ Some compute identification loss $\mathcal{L}_{id}(\mathcal{B}_{L}; \mathbf{\Theta}, \{\mathbf{p}_{c}\})$ in Eq. (5) Update cluster relation matrix $\hat{\mathbf{R}}^{dyn}$ by Eq. (8) $\hat{\mathbf{R}}^{dyn(t+1)} = \hat{\mathbf{R}}^{dyn(t)} - \gamma_{R} \frac{\partial \mathcal{L}_{id}(\mathcal{B}_{L}; \mathbf{\Theta}', \{\mathbf{p}_{c}'\})}{\partial \hat{\mathbf{R}}^{dyn(t)}}$ Update model parameter $\mathbf{\Theta}$ and prototype $\mathbf{p}_{c}$ with updated cluster relation matrix $\hat{\mathbf{R}}^{dyn(t+1)}$ by Eq. (9) and Eq. (10) $\mathbf{\Theta}^{(t+1)} = \mathbf{\Theta}^{(t)} - \gamma_{\theta} \frac{\partial \mathcal{L}_{cd}^{dyn}(\mathcal{B}_{U}; \hat{\mathbf{R}}^{dyn(t+1)}, \mathbf{\Theta}^{(t)}, \{\mathbf{p}_{c}^{(t)}\})}{\partial \mathbf{p}_{c}^{(t)}}$ $\mathbf{p}_{c}^{(t+1)} = \mathbf{p}_{c}^{(t)} - \gamma_{p} \frac{\partial \mathcal{L}_{cd}^{dyn}(\mathcal{B}_{U}; \hat{\mathbf{R}}^{dyn(t+1)}, \mathbf{\Theta}^{(t)}, \{\mathbf{p}_{c}^{(t)}\})}{\partial \mathbf{p}_{c}^{(t)}}$ 11 $t \leftarrow t + 1$ 12 end	7 Approximate optimal model parameter $\Theta'$ and
$ \begin{aligned} \Theta' &= \Theta^{(t)} - \alpha \frac{\partial \mathcal{L}_{cd}^{uyn}(\mathcal{B}_{U}; \mathbf{\hat{R}}^{uyn(t)}, \Theta^{(t)}, \{\mathbf{p}_{c}^{(t)}\})}{\partial \Theta^{(t)}} \\ \mathbf{p}_{c}' &= \mathbf{p}_{c}^{(t)} - \alpha \frac{\partial \mathcal{L}_{cd}^{dyn}(\mathcal{B}_{U}; \mathbf{\hat{R}}^{dyn(t)}, \Theta^{(t)}, \{\mathbf{p}_{c}^{(t)}\})}{\partial \mathbf{p}_{c}^{(t)}} \\ \mathbf{s} & \text{Compute identification loss } \mathcal{L}_{id}(\mathcal{B}_{L}; \Theta, \{\mathbf{p}_{c}\}) \text{ in } \\ \text{Eq. (5)} \\ \mathbf{y} & \text{Update cluster relation matrix } \mathbf{\hat{R}}^{dyn} \text{ by Eq. (8)} \\ \mathbf{\hat{R}}^{dyn(t+1)} &= \mathbf{\hat{R}}^{dyn(t)} - \gamma_{R} \frac{\partial \mathcal{L}_{id}(\mathcal{B}_{L}; \Theta', \{\mathbf{p}_{c}'\})}{\partial \mathbf{\hat{R}}^{dyn(t)}} \\ \text{Update model parameter } \Theta \text{ and prototype } \mathbf{p}_{c} \\ \text{with updated cluster relation matrix } \mathbf{\hat{R}}^{dyn(t+1)} \\ \text{by Eq. (9) and Eq. (10)} \\ \Theta^{(t+1)} &= \Theta^{(t)} - \gamma_{\theta} \frac{\partial \mathcal{L}_{cd}^{dyn}(\mathcal{B}_{U}; \mathbf{\hat{R}}^{dyn(t+1)}, \Theta^{(t)}, \{\mathbf{p}_{c}^{(t)}\})}{\partial \mathbf{p}_{c}^{(t)}} \\ \mathbf{p}_{c}^{(t+1)} &= \mathbf{p}_{c}^{(t)} - \gamma_{p} \frac{\partial \mathcal{L}_{cd}^{dyn}(\mathcal{B}_{U}; \mathbf{\hat{R}}^{dyn(t+1)}, \Theta^{(t)}, \{\mathbf{p}_{c}^{(t)}\})}{\partial \mathbf{p}_{c}^{(t)}} \\ 10  t \leftarrow t+1 \\ 11 \text{ te end} \end{aligned}$	prototype parameter $\mathbf{p}_c'$ by Eq. (6) and Eq. (7)
$ \mathbf{p}_{c}' = \mathbf{p}_{c}^{(t)} - \alpha \frac{\partial \mathcal{L}_{cd}^{dyn}(\mathcal{B}_{U}; \hat{\mathbf{R}}^{dyn(t)}, \mathbf{\Theta}^{(t)}, \{\mathbf{p}_{c}^{(t)}\})}{\partial \mathbf{p}_{c}^{(t)}} $ Some compute identification loss $\mathcal{L}_{id}(\mathcal{B}_{L}; \mathbf{\Theta}, \{\mathbf{p}_{c}\})$ in Eq. (5) Update cluster relation matrix $\hat{\mathbf{R}}^{dyn}$ by Eq. (8) $\hat{\mathbf{R}}^{dyn(t+1)} = \hat{\mathbf{R}}^{dyn(t)} - \gamma_{R} \frac{\partial \mathcal{L}_{id}(\mathcal{B}_{L}; \mathbf{\Theta}', \{\mathbf{p}_{c}'\})}{\partial \hat{\mathbf{R}}^{dyn(t)}} $ Update model parameter $\mathbf{\Theta}$ and prototype $\mathbf{p}_{c}$ with updated cluster relation matrix $\hat{\mathbf{R}}^{dyn(t+1)}$ by Eq. (9) and Eq. (10) $\mathbf{\Theta}^{(t+1)} = \mathbf{\Theta}^{(t)} - \gamma_{\theta} \frac{\partial \mathcal{L}_{cd}^{dyn}(\mathcal{B}_{U}; \hat{\mathbf{R}}^{dyn(t+1)}, \mathbf{\Theta}^{(t)}, \{\mathbf{p}_{c}^{(t)}\})}{\partial \mathbf{\Theta}^{(t)}} $ $\mathbf{p}_{c}^{(t+1)} = \mathbf{p}_{c}^{(t)} - \gamma_{p} \frac{\partial \mathcal{L}_{cd}^{dyn}(\mathcal{B}_{U}; \hat{\mathbf{R}}^{dyn(t+1)}, \mathbf{\Theta}^{(t)}, \{\mathbf{p}_{c}^{(t)}\})}{\partial \mathbf{p}_{c}^{(t)}} $ 10 $t \leftarrow t + 1$ 12 end	$\boldsymbol{\Theta}' = \boldsymbol{\Theta}^{(t)} - \alpha \frac{\partial \mathcal{L}_{cd}^{uyn}(\mathcal{B}_U; \mathbf{R}^{uyn(t)}, \boldsymbol{\Theta}^{(t)}, \{\mathbf{p}_c^{(t)}\})}{\partial \boldsymbol{\Theta}^{(t)}}$
8 Compute identification loss $\hat{\mathcal{L}}_{id}(\mathcal{B}_L; \Theta, \{\mathbf{p}_c\})$ in Eq. (5) 9 Update cluster relation matrix $\hat{\mathbf{R}}^{dyn}$ by Eq. (8) $\hat{\mathbf{R}}^{dyn(t+1)} = \hat{\mathbf{R}}^{dyn(t)} - \gamma_R \frac{\partial \mathcal{L}_{id}(\mathcal{B}_L; \Theta', \{\mathbf{p}_c\})}{\partial \hat{\mathbf{R}}^{dyn(t)}}$ 10 Update model parameter $\Theta$ and prototype $\mathbf{p}_c$ with updated cluster relation matrix $\hat{\mathbf{R}}^{dyn(t+1)}$ by Eq. (9) and Eq. (10) $\Theta^{(t+1)} = \Theta^{(t)} - \gamma_{\theta} \frac{\partial \mathcal{L}_{cd}^{dyn}(\mathcal{B}_U; \hat{\mathbf{R}}^{dyn(t+1)}, \Theta^{(t)}, \{\mathbf{p}_c^{(t)}\})}{\partial \Theta^{(t)}}$ $\mathbf{p}_c^{(t+1)} = \mathbf{p}_c^{(t)} - \gamma_p \frac{\partial \mathcal{L}_{cd}^{dyn}(\mathcal{B}_U; \hat{\mathbf{R}}^{dyn(t+1)}, \Theta^{(t)}, \{\mathbf{p}_c^{(t)}\})}{\partial \mathbf{p}_c^{(t)}}$ 11 $t \leftarrow t+1$ 12 end	$\mathbf{p}_{c}' = \mathbf{p}_{c}^{(t)} - \alpha \frac{\partial \mathcal{L}_{cd}^{dyn}(\mathcal{B}_{U}; \hat{\mathbf{R}}^{dyn(t)}, \boldsymbol{\Theta}^{(t)}, \{\mathbf{p}_{c}^{(t)}\})}{\alpha}$
10 Eq. (5) 10 Eq. (5) 10 Update cluster relation matrix $\hat{\mathbf{R}}^{dyn}$ by Eq. (8) $\hat{\mathbf{R}}^{dyn(t+1)} = \hat{\mathbf{R}}^{dyn(t)} - \gamma_R \frac{\partial \mathcal{L}_{id}(\mathcal{B}_L; \Theta', \{\mathbf{p}'_c\})}{\partial \hat{\mathbf{R}}^{dyn(t)}}$ 10 Update model parameter $\Theta$ and prototype $\mathbf{p}_c$ with updated cluster relation matrix $\hat{\mathbf{R}}^{dyn(t+1)}$ by Eq. (9) and Eq. (10) $\Theta^{(t+1)} = \Theta^{(t)} - \gamma_{\theta} \frac{\partial \mathcal{L}_{cd}^{dyn}(\mathcal{B}_U; \hat{\mathbf{R}}^{dyn(t+1)}, \Theta^{(t)}, \{\mathbf{p}_c^{(t)}\})}{\partial \Theta^{(t)}}$ $\mathbf{p}_c^{(t+1)} = \mathbf{p}_c^{(t)} - \gamma_p \frac{\partial \mathcal{L}_{cd}^{dyn}(\mathcal{B}_U; \hat{\mathbf{R}}^{dyn(t+1)}, \Theta^{(t)}, \{\mathbf{p}_c^{(t)}\})}{\partial \mathbf{p}_c^{(t)}}$ 11 $t \leftarrow t+1$ 12 end	8 Compute identification loss $\int_{\mathcal{A}} (\mathcal{B}_T \cdot \Theta \{\mathbf{p}_n\})$ in
9 Update cluster relation matrix $\hat{\mathbf{R}}^{dyn}$ by Eq. (8) $\hat{\mathbf{R}}^{dyn(t+1)} = \hat{\mathbf{R}}^{dyn(t)} - \gamma_R \frac{\partial \mathcal{L}_{id}(\mathcal{B}_L; \mathbf{\Theta}', \{\mathbf{p}_c'\})}{\partial \hat{\mathbf{R}}^{dyn(t)}}$ 10 Update model parameter $\mathbf{\Theta}$ and prototype $\mathbf{p}_c$ with updated cluster relation matrix $\hat{\mathbf{R}}^{dyn(t+1)}$ by Eq. (9) and Eq. (10) $\mathbf{\Theta}^{(t+1)} = \mathbf{\Theta}^{(t)} - \gamma_{\theta} \frac{\partial \mathcal{L}_{cd}^{dyn}(\mathcal{B}_U; \hat{\mathbf{R}}^{dyn(t+1)}, \mathbf{\Theta}^{(t)}, \{\mathbf{p}_c^{(t)}\})}{\partial \mathbf{\Theta}^{(t)}}$ $\mathbf{p}_c^{(t+1)} = \mathbf{p}_c^{(t)} - \gamma_p \frac{\partial \mathcal{L}_{cd}^{dyn}(\mathcal{B}_U; \hat{\mathbf{R}}^{dyn(t+1)}, \mathbf{\Theta}^{(t)}, \{\mathbf{p}_c^{(t)}\})}{\partial \mathbf{p}_c^{(t)}}$ 11 $t \leftarrow t+1$ 12 end	Eq. (5)
$ \hat{\mathbf{R}}^{dyn(t+1)} = \hat{\mathbf{R}}^{dyn(t)} - \gamma_R \frac{\partial \mathcal{L}_{id}(\mathcal{B}_L; \Theta', \{\mathbf{p}'_c\})}{\partial \hat{\mathbf{R}}^{dyn(t)}} \\ \text{Update model parameter } \Theta \text{ and prototype } \mathbf{p}_c \\ \text{with updated cluster relation matrix } \hat{\mathbf{R}}^{dyn(t+1)} \\ \text{by Eq. (9) and Eq. (10)} \\ \Theta^{(t+1)} = \Theta^{(t)} - \gamma_\theta \frac{\partial \mathcal{L}_{cd}^{dyn}(\mathcal{B}_U; \hat{\mathbf{R}}^{dyn(t+1)}, \Theta^{(t)}, \{\mathbf{p}_c^{(t)}\})}{\partial \Theta^{(t)}} \\ \mathbf{p}_c^{(t+1)} = \mathbf{p}_c^{(t)} - \gamma_p \frac{\partial \mathcal{L}_{cd}^{dyn}(\mathcal{B}_U; \hat{\mathbf{R}}^{dyn(t+1)}, \Theta^{(t)}, \{\mathbf{p}_c^{(t)}\})}{\partial \mathbf{p}_c^{(t)}} \\ \mathbf{t} \leftarrow t+1 \\ \text{12 end} $	9 Update cluster relation matrix $\hat{\mathbf{R}}^{dyn}$ by Eq. (8)
10 Update model parameter $\Theta$ and prototype $\mathbf{p}_{c}$ with updated cluster relation matrix $\hat{\mathbf{R}}^{dyn(t+1)}$ by Eq. (9) and Eq. (10) $\Theta^{(t+1)} = \Theta^{(t)} - \gamma_{\theta} \frac{\partial \mathcal{L}_{cd}^{dyn}(\mathcal{B}_{U}; \hat{\mathbf{R}}^{dyn(t+1)}, \Theta^{(t)}, \{\mathbf{p}_{c}^{(t)}\})}{\partial \Theta^{(t)}}$ $\mathbf{p}_{c}^{(t+1)} = \mathbf{p}_{c}^{(t)} - \gamma_{p} \frac{\partial \mathcal{L}_{cd}^{dyn}(\mathcal{B}_{U}; \hat{\mathbf{R}}^{dyn(t+1)}, \Theta^{(t)}, \{\mathbf{p}_{c}^{(t)}\})}{\partial \mathbf{p}_{c}^{(t)}}$ 11 $t \leftarrow t+1$ 12 end	$\hat{\mathbf{R}}^{dyn(t+1)} = \hat{\mathbf{R}}^{dyn(t)} - \gamma_R \frac{\partial \mathcal{L}_{id}(\mathcal{B}_L; \mathbf{\Theta}', \{\mathbf{p}_c'\})}{\partial \hat{\boldsymbol{\rho}}^{dyn(t)}}$
with updated cluster relation matrix $\hat{\mathbf{R}}^{dyn(t+1)}$ by Eq. (9) and Eq. (10) $\boldsymbol{\Theta}^{(t+1)} = \boldsymbol{\Theta}^{(t)} - \gamma_{\theta} \frac{\partial \mathcal{L}_{cd}^{dyn}(\mathcal{B}_{U}; \hat{\mathbf{R}}^{dyn(t+1)}, \boldsymbol{\Theta}^{(t)}, \{\mathbf{p}_{c}^{(t)}\})}{\mathbf{p}_{c}^{(t+1)}} = \mathbf{p}_{c}^{(t)} - \gamma_{p} \frac{\partial \mathcal{L}_{cd}^{dyn}(\mathcal{B}_{U}; \hat{\mathbf{R}}^{dyn(t+1)}, \boldsymbol{\Theta}^{(t)}, \{\mathbf{p}_{c}^{(t)}\})}{\partial \mathbf{p}_{c}^{(t)}}$ 11 $t \leftarrow t+1$ 12 end	10 Update model parameter $\Theta$ and prototype $\mathbf{p}_c$
by Eq. (9) and Eq. (10) $\Theta^{(t+1)} = \Theta^{(t)} - \gamma_{\theta} \frac{\partial \mathcal{L}_{cd}^{dyn}(\mathcal{B}_{U}; \hat{\mathbf{R}}^{dyn(t+1)}, \Theta^{(t)}, \{\mathbf{p}_{c}^{(t)}\})}{\partial \Theta^{(t)}}$ $\mathbf{p}_{c}^{(t+1)} = \mathbf{p}_{c}^{(t)} - \gamma_{p} \frac{\partial \mathcal{L}_{cd}^{dyn}(\mathcal{B}_{U}; \hat{\mathbf{R}}^{dyn(t+1)}, \Theta^{(t)}, \{\mathbf{p}_{c}^{(t)}\})}{\partial \mathbf{p}_{c}^{(t)}}$ $t \leftarrow t+1$ 12 end	with updated cluster relation matrix $\hat{\mathbf{R}}^{dyn(t+1)}$
$ \begin{aligned} \mathbf{\Theta}^{(t+1)} &= \mathbf{\Theta}^{(t)} - \gamma_{\theta} \frac{\partial \mathcal{L}_{cd}^{dyn}(\mathcal{B}_{U}; \hat{\mathbf{R}}^{dyn(t+1)}, \mathbf{\Theta}^{(t)}, \{\mathbf{p}_{c}^{(t)}\})}{\partial \mathbf{\Theta}^{(t)}} \\ \mathbf{p}_{c}^{(t+1)} &= \mathbf{p}_{c}^{(t)} - \gamma_{p} \frac{\partial \mathcal{L}_{cd}^{dyn}(\mathcal{B}_{U}; \hat{\mathbf{R}}^{dyn(t+1)}, \mathbf{\Theta}^{(t)}, \{\mathbf{p}_{c}^{(t)}\})}{\partial \mathbf{p}_{c}^{(t)}} \\ t \leftarrow t+1 \\ \text{12 end} \end{aligned} $	by Eq. (9) and Eq. (10)
$\mathbf{p}_{c}^{(t+1)} = \mathbf{p}_{c}^{(t)} - \gamma_{p} \frac{\partial \mathcal{L}_{cd}^{dyn}(\mathcal{B}_{U}; \hat{\mathbf{R}}^{dyn(t+1)}, \mathbf{\Theta}^{(t)}, \{\mathbf{p}_{c}^{(t)}\})}{\partial \mathbf{p}_{c}^{(t)}}$ $t \leftarrow t + 1$ 12 end	$\boldsymbol{\Theta}^{(t+1)} = \boldsymbol{\Theta}^{(t)} - \gamma_{\theta} \frac{\partial \mathcal{L}_{cd}^{dyn}(\mathcal{B}_{U}; \hat{\mathbf{R}}^{dyn(t+1)}, \boldsymbol{\Theta}^{(t)}, \{\mathbf{p}_{c}^{(t)}\})}{\partial \mathbf{Q}^{(t)}}$
11 $  t \leftarrow t+1$ 12 end	$\mathbf{p}_{c}^{(t+1)} = \mathbf{p}_{c}^{(t)} - \gamma_{p} \frac{\partial \mathcal{L}_{cd}^{dyn}(\mathcal{B}_{U}; \hat{\mathbf{R}}^{dyn(t+1)}, \mathbf{\Theta}^{(t)}, \{\mathbf{p}_{c}^{(t)}\})}{2}$
12 end	$11  t \leftarrow t+1 \qquad \qquad$
	12 end

## 2 MORE VISUALIZATIONS

**Visualization of Entropy Distribution.** To better understand the effect of reducing uncertainty of the underlying sample relations, we quantify the uncertainty by entropy for



Fig. S1. Entropy distributions of our method  $R^2D_{aff}$ , clustering-based pseudo label training (Cluster<sub>aff</sub>) and direct transfer.

our method R<sup>2</sup>D<sub>aff</sub>, rewarded relation discovery based on affinity-based cluster construction. To quantify the sample relations on DukeMTMC [1], we exploit the ground-truth attribute annotations [2] of outfit colors of upper body and lower body, including black, white, red, purple, gray, blue, green, brown of upper body and black, white, red, gray, blue, green, brown of lower body. These attributes are represented by a 15-dimensional vector. We regard the samples with the same attribute vector as samples of the same pseudo class. We expect that the underlying sample relations satisfy that the intra-pseudo-class similarity should be larger than inter-pseudo-class similarity. For each sample, the pseudo class probabilities are computed by a Softmax classifier parameterized by pseudo class center features, i.e., mean features of each pseudo class normalized by L2-norm. We quantify the uncertainty of the underlying sample relations by the entropy of pseudo class probabilities and show the entropy distribution of the training set of DukeMTMC.

Besides our method  $R^2D_{aff}$ , we compare the entropy distributions of clustering-based pseudo label training (Cluster<sub>aff</sub>) and direct transfer. Cluster<sub>aff</sub> uses affinity-based clusters for pseudo label training on unlabeled data without reward. Direct transfer is a baseline method of testing the pretrained model. The entropy distribution comparison is shown in Figure S1.

Compared with direct transfer, our method  $R^2D_{aff}$  reduces entropy more significantly than  $Cluster_{aff}$  learned



Fig. S2. Visualization of probabilistic relation between randomly selected query images and clusters. Each cluster is represented by an image in it and the value on the top of the image is the Cosine distance between the query image and cluster center. In each two rows, the distances in the first/second row are computed by model before/after using our method R<sup>2</sup>D. In sub-figure (a)/(b), the green/red arrows show the cluster relation changes that bring improvement/degradation. For the normal query images in sub-figure (a), the feature similarities become more consistent with visual similarities of human perception. For the query samples with occlusions or non-target persons in sub-figure (b), the interferences cause cluster relation changes that are contradictory to visual similarities of human perception. The cases of degradation are only in the minority.

without reward for updating the pseudo labels, which demonstrates the effectiveness of uncertainty reduction by our rewarded relation discovery method.

**Visualization of Learned Probabilistic Relations.** In section 6.5.5 in the main manuscript, we visualize some randomly selected clusters and the cluster relation changes after using our method  $R^2D_{aff}$ . For more comprehensive understanding of the effect of our method, we provide more examples of normal cases and hard cases on MSMT17-NA [3]. We randomly select query images and 10 clusters to visualize the Cosine distances between them in Figure S2. Each cluster

is represented by an image in it and the value on the top of the image is the Cosine distance between the query image and cluster center. In each two rows, the cluster relations before and after using our method are shown in the first row and the second row, respectively. In Figure S2 (a), the green arrows show the cluster relation changes that bring improvement; in Figure S2 (b), the red arrows show the cluster relation changes that bring degradation.

It can be observed that, in the normal cases shown in Figure S2 (a), our method can make the features of visually similar clusters closer to the features of query images. The cluster relations become more consistent with human perception; in the hard cases shown in Figure S2 (b), our method wrongly increases the feature similarities between the query image and some visually dissimilar clusters, because the target query person is occluded by other objects or other non-target pedestrians. Note that, in the first three groups of Figure S2 (b), the appearances of the topranking clusters are similar to those of occlusions or nontarget persons in the query image. In the training set, these hard cases are only in the minority. Thus, our method can learn to improve the cluster relations in most normal cases and better quantify the uncertainty of underlying sample relations.

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