Week 8: Generative Adversarial Networks

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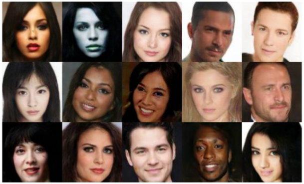
1. Generative adversarial network

2. Wasserstein GANs

Generative model

- Objective: generate new data by learning from training data
- Various applications: realistic new designs, super-resolution, colorization, etc.

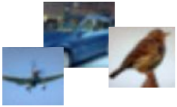


- Data augmentation: more realistic data for model training
- Data mining: exploring latent structure/representation of data

Figures here and in the next 7 slides from Stanford CS231n 2017 Lecture 13;
also see Goodfellow et al., "Generative adversarial nets", NIPS, 2014

Generative model (cont')

- What to learn from training data?
  One way is to learn data's density distribution from training data, then generate new data from the learned distribution.



Training data ~ $p_{data}(x)$

Generated samples ~ $p_{model}(x)$

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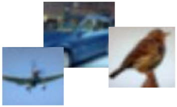

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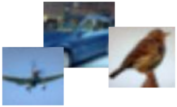

Training data ~ $p_{data}(x)$

Generated samples ~ $p_{model}(x)$

- Difficulty: never know real data complex distribution $P_{data}(x)$, therefore difficult to learn its approximation $P_{model}(x)$
- Solution: learn to generate new data directly, without learning the data distribution!

Generative model (cont')

- Problem: How to generate data without learning complex high-dim (image) data distribution?

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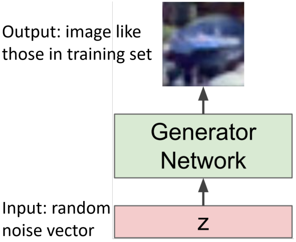
Generative model (cont')

- Problem: How to generate data without learning complex high-dim (image) data distribution?
- Idea: sample from simple distribution (e.g., of random noise), and learn a complex transformation from simple distribution to the (unknown) complex distribution of image data.
- If using a neural network to represent the transformation, the objective is to train the network to generate realistic images whose distribution is similar to that of training dataset.



Generative adversarial network (GAN): a two-player game

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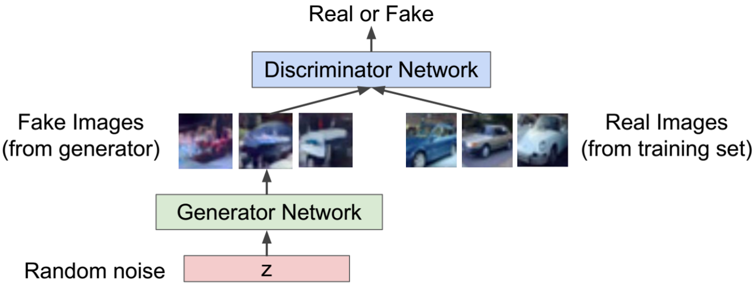
- Key insight: Use another network to help evaluate it!



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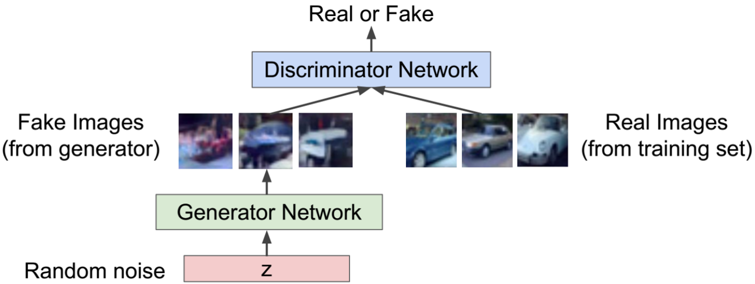

- G network: try to fool D by generating realistic images
- D network: try to distinguish between real and fake images

GAN training

- The two objectives can be achieved by

$$\min_{G_\theta} \max_{D_w} \{\mathbb{E}_{x \sim P_r} [\log D_w(x)] + \mathbb{E}_{z \sim P(z)} [\log(1 - D_w(G_\theta(z)))]\}$$

where discriminator network D_w is a binary classifier.

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Train the two networks alternatively:

1: Training D network, i.e. $\max_{D_w}\{\cdot\}$, makes $D_w(x)$ close to 1 for real images and $D_w(G(z))$ close to 0 for fake images $G(z)$

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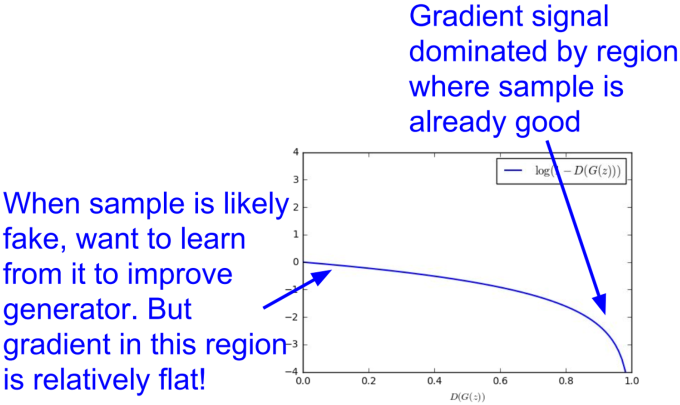
2: Training G network, i.e. $\min_{G_\theta}\{\cdot\}$, makes $D_w(G(z))$ close to 1, fooling D into thinking $G(z)$ is real.

$$\min_{G_\theta} \{\mathbb{E}_{z\sim P(z)}\left[\log(1 - D_w(G_\theta(z)))\right]\}$$

GAN training (cont')

- However, optimizing generator's objective does not work well!
- Figure: x-axis for $D(G(z))$ and y-axis for $\log(1 - D(G(z)))$

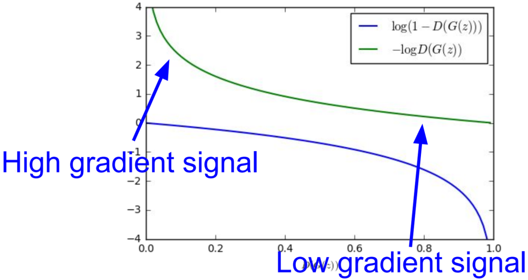


GAN training (cont')

- Instead, optimizing generator by

$$\min_{G_\theta}\{\mathbb{E}_{z\sim P(z)}\left[-\log(D_w(G_\theta(z)))\right]\}$$

which also tries to fool discriminator, but now with higher gradient signal for poor generator.



GAN implementation

for number of training iterations **do**

for k steps **do**

- Sample minibatch of m noise samples $\{\boldsymbol{z}^{(1)}, \ldots, \boldsymbol{z}^{(m)}\}$ from noise prior $p_g(\boldsymbol{z})$.
- Sample minibatch of m examples $\{\boldsymbol{x}^{(1)}, \ldots, \boldsymbol{x}^{(m)}\}$ from data generating distribution $p_{\text{data}}(\boldsymbol{x})$.
- Update the discriminator by ascending its stochastic gradient:

$$\nabla_{\theta_d} \frac{1}{m} \sum_{i=1}^{m} \left[\log D_{\theta_d}(x^{(i)}) + \log(1 - D_{\theta_d}(G_{\theta_g}(z^{(i)}))) \right]$$

Update Discriminator

end for

- Sample minibatch of m noise samples $\{\boldsymbol{z}^{(1)}, \ldots, \boldsymbol{z}^{(m)}\}$ from noise prior $p_g(\boldsymbol{z})$.
- Update the generator by ascending its stochastic gradient (improved objective):

$$\nabla_{\theta_g} \frac{1}{m} \sum_{i=1}^{m} \log(D_{\theta_d}(G_{\theta_g}(z^{(i)})))$$

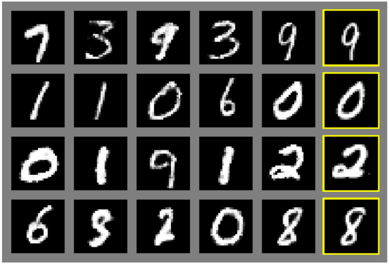
Update Generator

end for

- Note: $D_{\theta_d} = D_w$ and $G_{\theta_g} = G_\theta$

GAN result

- Generator (consisting of FC layers) generated realistic images
- Generator does not simply remember training images



- Yellow boxes: nearest training example of neighboring generated sample

Figure from Goodfellow et al., "Generative adversarial nets", NIPS, 2014

DCGAN: deep convolutional GAN

- Generator is a 'deconvolutional' network
- Discriminator is a convolutional network
- It is not that easy to make DCGAN work!

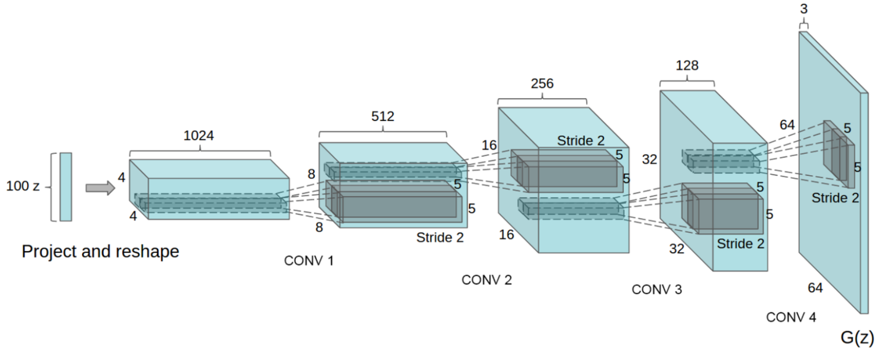
Architecture guidelines for stable Deep Convolutional GANs

- Replace any pooling layers with strided convolutions (discriminator) and fractional-strided convolutions (generator).
- Use batchnorm in both the generator and the discriminator.
- Remove fully connected hidden layers for deeper architectures.
- Use ReLU activation in generator for all layers except for the output, which uses Tanh.
- Use LeakyReLU activation in the discriminator for all layers.

Table here and figures in next 5 slides from Radford, Metz, Chintala, "Unsupervised representation learning with deep convolutional generative adversarial networks", ICLR, 2016

DCGAN (cont')

- The generator's structure



DCGAN result

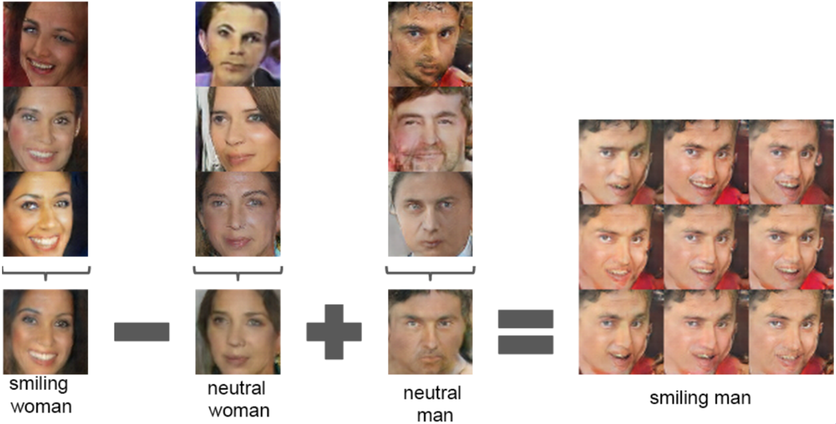
- Generated images are already realistic after one epoch training

DCGAN result (cont')

- Interpolations between random points in the latent (input) space result in semantic changes in generated images

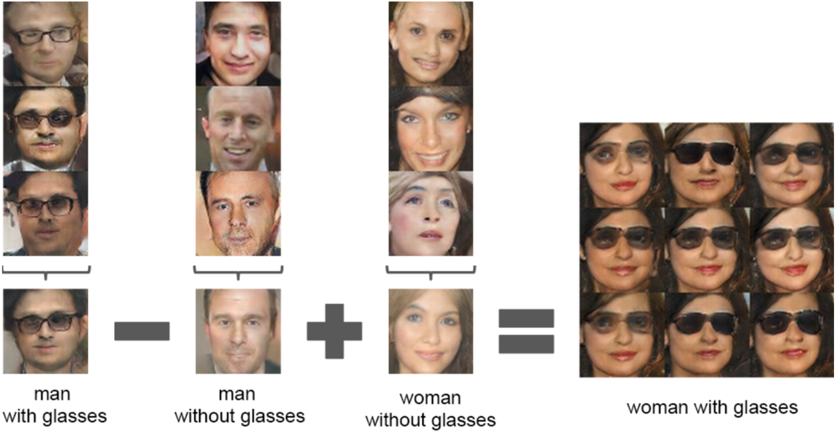
DCGAN result (cont'): interpretable vector arithmetic

In latent space:
mean smiling woman vector - mean neural woman vector + mean neural man vector = mean smiling man vector!



DCGAN result (cont'): interpretable vector arithmetic

- Similarly, women-with-glass images can be generated by vector arithmetic in the latent space.



LSGAN: Least Square Generative Adversarial Networks

- Uses least-square loss to relieve gradient vanishing issue. Original GAN used (sigmoid) binary cross-entropy loss.
- Tries to achieve the same objective as original GAN

Original GAN

$$\min_G \max_D V_{\text{GAN}}(D,G) = \mathbb{E}_{\boldsymbol{x}\sim p_{\text{data}}(\boldsymbol{x})}[\log D(\boldsymbol{x})] + \mathbb{E}_{\boldsymbol{z}\sim p_{\boldsymbol{z}}(\boldsymbol{z})}[\log(1-D(G(\boldsymbol{z})))]$$

LS-GAN

$$\min_D V_{\text{LSGAN}}(D) = \frac{1}{2}\mathbb{E}_{\boldsymbol{x}\sim p_{\text{data}}(\boldsymbol{x})}\left[(D(\boldsymbol{x})-1)^2\right] + \frac{1}{2}\mathbb{E}_{\boldsymbol{z}\sim p_{\boldsymbol{z}}(\boldsymbol{z})}\left[(D(G(\boldsymbol{z})))^2\right]$$

$$\min_G V_{\text{LSGAN}}(G) = \frac{1}{2}\mathbb{E}_{\boldsymbol{z}\sim p_{\boldsymbol{z}}(\boldsymbol{z})}\left[(D(G(\boldsymbol{z}))-1)^2\right].$$

Formulae here and figures in next slide from Mao, Li, Xie, Lau, Wang, Smolley, "Least squares generative adversarial networks", ICCV, 2017

LSGAN result

- LSGAN generated good-quality (e.g., outdoor and indoor) images


- LSGAN is more stable than regular GAN, when batch normalization is removed in generator.

(a) LSGANs.

(b) Regular GANs.

Issues in GAN training

- Unstable training: when Discriminator becomes perfect, loss becomes zero, so gradient for Generator vanishes!

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- Unstable training: when Discriminator becomes perfect, loss becomes zero, so gradient for Generator vanishes!
  On the other hand, if Discriminator behave badly, Generator would have bad feedback and cannot update well.
- Mode collapse: generator generates realistic images, but with low varieties

- Lack of proper evaluation metric: do not know when to stop training.

See theoretical proof behind the issues from Arjovsky, Bottou, "Towards principled methods for training generative adversarial networks", arXiv, 2017

Essentially,...

- Generator is used to generate fake data whose distribution P_θ approximates real but unknown data distribution P_r
- Discriminator helps measures difference between P_r and P_θ (using loss function of GAN based on discriminator's output)

Any better way to measure difference between distributions?

Distance measurements between two distributions

- Kullback-Leibler (KL) divergence

$$KL(P_r \parallel P_\theta) \quad = \quad \int P_r(x) \log \left(\frac{P_r(x)}{P_\theta(x)}\right) dx$$

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- Jenson-Shannon (JS) divergence, with $P_m = (P_r + P_\theta)/2$:

$$JS(P_r, P_\theta) = \frac{1}{2} KL(P_r \parallel P_m) + \frac{1}{2} KL(P_\theta \parallel P_m)$$

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- Earth Mover's distance or Wasserstein distance:

$$W(P_r, P_\theta) = \inf_{\gamma \in \Pi(P_r, P_\theta)} \mathbb{E}_{(x,y)\sim\gamma} [\|x - y\|]$$

where $\Pi(P_r, P_\theta)$ is the set of all joint distributions whose marginal distributions are P_r and P_θ.

Earth Mover's Distance (EMD)

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- 'work': amount of moved earth in a chunk times the distance moved

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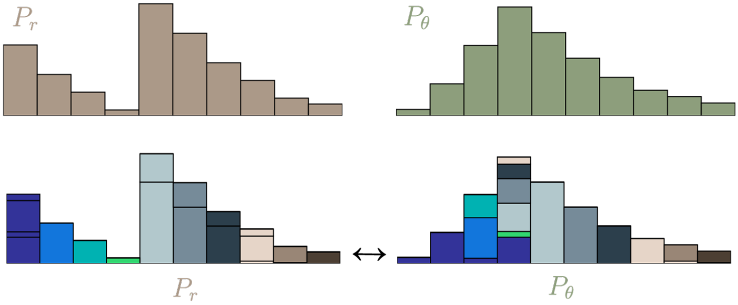
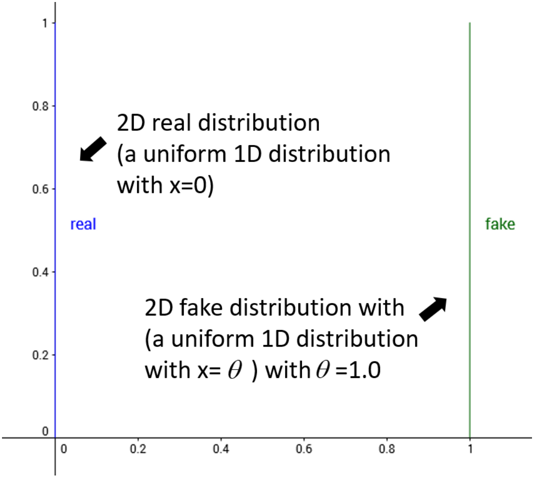


Figures from https://vincentherrmann.github.io/blog/wasserstein/

EMD or Wasserstein distance

- Example of measuring the distance between real distribution P_0 (i.e., P_r) and fake distribution P_θ



EMD or Wasserstein distance (cont')

- $W(\mathbb{P}_0, \mathbb{P}_\theta) = |\theta|$,
- $JS(\mathbb{P}_0, \mathbb{P}_\theta) = \begin{cases} \log 2 & \text{if } \theta \neq 0 \ , \\ 0 & \text{if } \theta = 0 \ , \end{cases}$
- $KL(\mathbb{P}_\theta \| \mathbb{P}_0) = KL(\mathbb{P}_0 \| \mathbb{P}_\theta) = \begin{cases} +\infty & \text{if } \theta \neq 0 \ , \\ 0 & \text{if } \theta = 0 \ , \end{cases}$

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- The gradient of JS and KL divergence (over θ) is always 0, therefore failing to update θ.
- Wasserstein distance is continuous and has non-zero gradient almost everywhere
- Therefore, JS and KL divergences are not good choices to measure the distance between two distributions.

Wasserstein distance

- It is intractable to compute Wasserstein distance exactly

$$W(P_r, P_g) \quad = \quad \inf_{\gamma \in \Pi(P_r, P_g)} \mathbb{E}_{(x,y)\sim\gamma} \left[\|x - y\| \right]$$

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- But it shows W is equivalent to

$$W(P_r, P_\theta) = \sup_{\|f\|_L \leq 1} \mathbb{E}_{x\sim P_r} \left[f(x) \right] - \mathbb{E}_{x\sim P_\theta} \left[f(x) \right]$$

where $\|f\|_L \leq 1$ means f is a 1-Lipschitz function.

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  where $\|f\|_L \leq 1$ means f is a 1-Lipschitz function.
- Let d_X and d_Y be distance functions on spaces X and Y. A function $f : X \to Y$ is K-Lipschitz if for all $x_1, x_2 \in X$,

$$d_Y(f(x_1), f(x_2)) \leq K d_X(x_1, x_2)$$

  Intuitively: if x_1, x_2 are close, $f(x_1)$ and $f(x_2)$ are also close.

Wasserstein distance approximation

- The supremum over 1-Lipschitz functions is still intractable

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- Suppose there are a parameterized function family $\{f_w\}_{w\in\mathcal{W}}$ with parameter w and function f_w being K-Lipschitz, then

$$\max_{w\in\mathcal{W}} \mathbb{E}_{x\sim P_r}[f_w(x)] - \mathbb{E}_{x\sim P_\theta}[f_w(x)]$$
$$\leq \sup_{\|f\|_L \leq K} \mathbb{E}_{x\sim P_r}[f(x)] - \mathbb{E}_{x\sim P_\theta}[f(x)]$$

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&= K \cdot W(P_r, P_\theta)
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\end{aligned}$$

- Then $W(P_r, P_\theta)$ can be approximated by

$$\max_{w\in\mathcal{W}} \mathbb{E}_{x\sim P_r}[f_w(x)] - \mathbb{E}_{x\sim P_\theta}[f_w(x)]$$

Note: K can be absorbed into learning rate during max.

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- Answer: weight clamping! Clipping w to be within $[-c, c]$ after every update of w.

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- Go back to original objective: In order to train Generator g_θ such that the fake $P_\theta = g_\theta(Z)$ matches real P_r, we need a distance measurement to estimate the difference between P_θ and P_r. Here the distance measurement is Wasserstein distance which can be approximately computed based on the optimal f_w, i.e., by optimizing a network work f_w. We call such a network f_w 'critic function', corresponding to Discriminator in original GAN.

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- Wasserstein GAN: generator + critic

Wasserstein GAN (cont')

- Once getting optimal f_w (by training critic network with maximization of $\mathbb{E}_{x\sim P_r}[f_w(x)] - \mathbb{E}_{x\sim P_\theta}[f_w(x)]$), then

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- Then update generator g_θ (with fixed f_w) by minimizing W

$$\begin{aligned}\nabla_\theta W(P_r, P_\theta) &= \nabla_\theta(\mathbb{E}_{x\sim P_r}[f_w(x)] - \mathbb{E}_{z\sim Z}[f_w(g_\theta(z))]) \\ &= -\mathbb{E}_{z\sim Z}[\nabla_\theta f_w(g_\theta(z))]\end{aligned}$$

Wasserstein GAN (cont')

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$$W(P_r, P_\theta) = \mathbb{E}_{x\sim P_r}[f_w(x)] - \mathbb{E}_{x\sim P_\theta}[f_w(x)]$$

- Then update generator g_θ (with fixed f_w) by minimizing W

$$\begin{aligned} \nabla_\theta W(P_r, P_\theta) &= \nabla_\theta(\mathbb{E}_{x\sim P_r}[f_w(x)] - \mathbb{E}_{z\sim Z}[f_w(g_\theta(z))]) \\ &= -\mathbb{E}_{z\sim Z}[\nabla_\theta f_w(g_\theta(z))] \end{aligned}$$

In summary, repeat the following steps to train WGAN:

Step 1: Fix generator g_θ, compute approximation of $W(P_r, P_\theta)$ by training f_w, i.e., by $\max_{w\in\mathcal{W}} \mathbb{E}_{x\sim P_r}[f_w(x)] - \mathbb{E}_{x\sim P_\theta}[f_w(x)]$

Step 2: Fix 'critic' f_w, update g_θ with gradient $-\mathbb{E}_{z\sim Z}[\nabla_\theta f_w(g_\theta(z))]$ by sampling several z from uniform distribution Z.

WGAN algorithm

Algorithm 1 WGAN, our proposed algorithm. All experiments in the paper used the default values $\alpha = 0.00005$, $c = 0.01$, $m = 64$, $n_{\text{critic}} = 5$.

Require: : α, the learning rate. c, the clipping parameter. m, the batch size. n_{critic}, the number of iterations of the critic per generator iteration.

Require: : w_0, initial critic parameters. θ_0, initial generator's parameters.

while θ has not converged **do**

 for $t = 0, ..., n_{\text{critic}}$ **do**

  Sample $\{x^{(i)}\}_{i=1}^m \sim \mathbb{P}_r$ a batch from the real data.

  Sample $\{z^{(i)}\}_{i=1}^m \sim p(z)$ a batch of prior samples.

  $g_w \leftarrow \nabla_w \left[\frac{1}{m}\sum_{i=1}^m f_w(x^{(i)}) - \frac{1}{m}\sum_{i=1}^m f_w(g_\theta(z^{(i)}))\right]$

  $w \leftarrow w + \alpha \cdot \text{RMSProp}(w, g_w)$

  $w \leftarrow \text{clip}(w, -c, c)$

 end for

 Sample $\{z^{(i)}\}_{i=1}^m \sim p(z)$ a batch of prior samples.

 $g_\theta \leftarrow -\nabla_\theta \frac{1}{m}\sum_{i=1}^m f_w(g_\theta(z^{(i)}))$

 $\theta \leftarrow \theta - \alpha \cdot \text{RMSProp}(\theta, g_\theta)$

end while

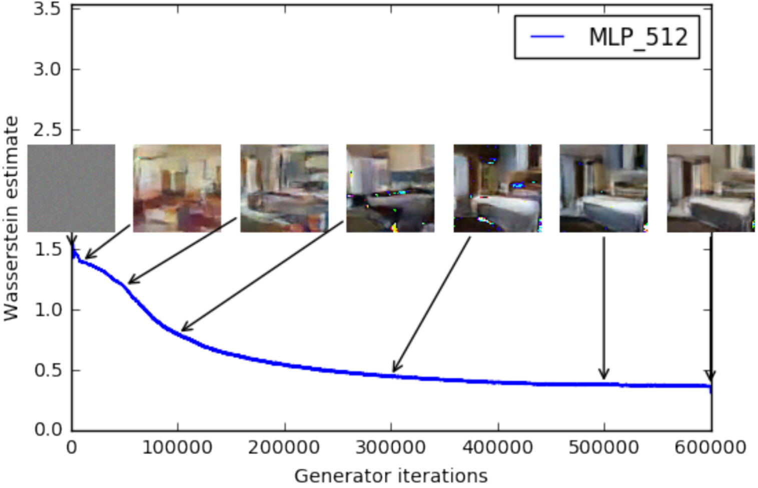
Step 1

Step 2

Table here and figures in the next 4 slides from Arjovsky, Chintala, Bottou, "Wasserstein GAN", arXiv, 2017

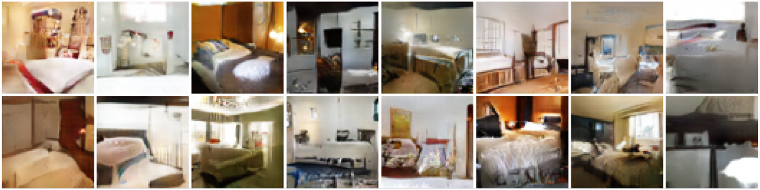
WGAN results

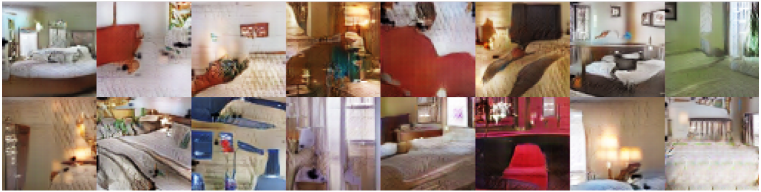
- Wasserstein loss correlates well with image quality



WGAN results (cont')

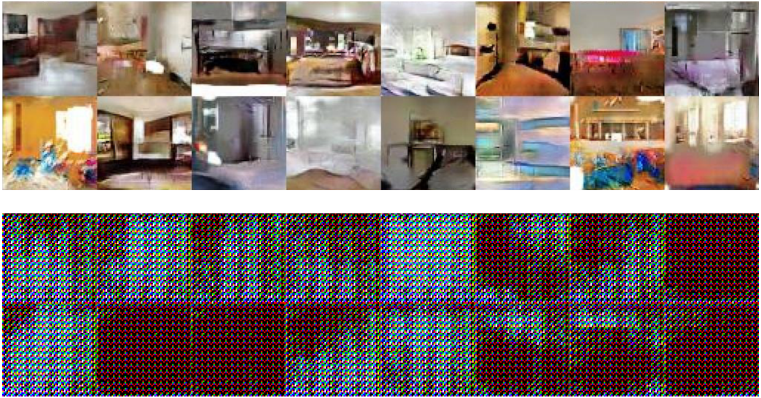
- WGAN (top 2 rows; with same DCGAN structure) performs well (if not better) compared to DCGAN (bottom 2 rows)





WGAN results (cont')

- WGAN (top 2 rows; with same DCGAN structure) still performs well when removing batch normalization, but DCGAN not (bottom 2 rows)



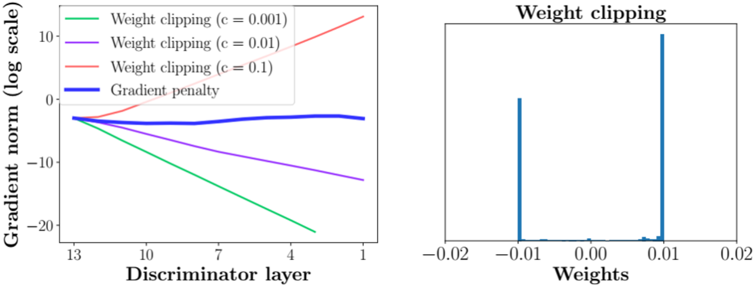
WGAN results (cont')

- WGAN (top 2 rows; with same MLP structure) reduce issue of mode collapse, compared to original GAN (bottom 2 rows).

However, ...

Issues of weight clipping in WGAN:

- makes critic f_w within a small subset of K-Lipschitz functions.
- causes gradient vanishing or exploding (Fig. left).
- pushes weights to extremes of clipping range (Fig. right), in turn causing gradient exploding and slow training.



Figures here and in the next 3 slides from Gulrajani, Ahmed, Arjovsky, Dumoulin, Courville, "Improved training of Wasserstein GANs", arXiv, 2017

WGAN-GP: GAN with gradient penalty

- Alternative way to enforce Lipschitz constraint: a function is 1-Lipschitz if and only if it has gradients (over input variable) with norm less or equal to 1.0 everywhere.

WGAN-GP: GAN with gradient penalty

- Alternative way to enforce Lipschitz constraint: a function is 1-Lipschitz if and only if it has gradients (over input variable) with norm less or equal to 1.0 everywhere.
- Critic loss (to be minimized)

$$
\begin{aligned}
L &= -\mathbb{E}_{x \sim P_r}\left[f_w(x)\right] + \mathbb{E}_{\tilde{x} \sim P_\theta}\left[f_w(\tilde{x})\right] + \\
&\quad \lambda \mathbb{E}_{\hat{x} \sim P_{\hat{x}}}\left[(\|\nabla_{\hat{x}} f_w(\hat{x})\|_2 - 1)^2\right]
\end{aligned}
$$

where the last term is 'gradient penalty' term, and

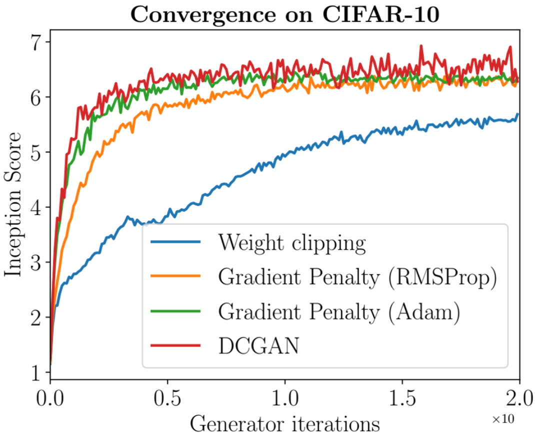
$$
\begin{aligned}
\epsilon \sim U[0,1], x \sim P_r, \tilde{x} \in P_\theta \\
\hat{x} = \epsilon x + (1-\epsilon)\tilde{x}
\end{aligned}
$$

considering optimal critic has gradient norm 1.0 on straight lines connecting points from P_r to P_θ.

- The improved WGAN is called WGAN-GP

WGAN-GP result

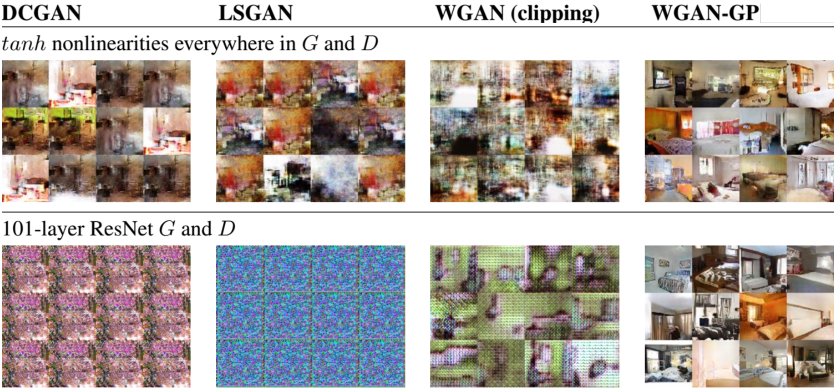
- WGAN-GP improves training speed compared to WGAN



- 'Inception score' is used to measure image quality

WGAN-GP result

- WGAN-GP successfully train difficult GAN architectures



Summary

- GANs provide an effective way to generate data
- GANs do not explicitly model data distribution
- GAN training: unstable and mode collapse
- WGAN: 1st attempt to improve GAN with theoretical proof
- More GANs already proposed, more GANs to be proposed!

Further reading:

- Miyato et al., Spectral normalization for generative adversarial networks, ICLR, 2018
- Dai et al., Good semi-supervised learning that requires a bad GAN, arXiv, 2017